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HEAT CONDUCTION IN SOLIDS WITH FINITE RATE OF DIFFUSION OF HEAT AND INITIAL CONDITIONS IN THE FORM OF RANDOM FUNCTIONS

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The heat-conduction problem in solids with finite rate of diffusion of heat and initial conditions in the form of a random functions is examined. The basic probabilistic characteristics of the process are obtained.

The behavior of real distributed objects under conditions of natural, industrial, and other noise are characterized by a certain uncertainty. The description of such systems with the help of well-known deterministic approaches it not always fruitful and often does not reflect the real picture. In studying heat-transfer processes, the circumstances indicated make it necessary to solve boundary-value problems for partial differential equations with a random right-hand side, random initial conditions, and random functions in the boundary conditions.

We shall examine a homogeneous isotropic planar layer of matter with thickness l. We shall assume that there are no internal heat sources in the layer, the rate of diffusion of heat is finite, and the initial state of the layer is described with the help of random spatial functions.

In order to determine the basic probabilistic characteristics of the temperature field in the layer, it is necessary to solve the following boundary-value problem:

$$\tau_r \frac{\partial^2 T(x,\ \tau)}{\partial \tau^2} + \frac{\partial T(x,\ \tau)}{\partial \tau} = a\frac{\partial^2 T(x,\ \tau)}{\partial x^2}, \tag{1}$$

$$T(x,\ 0) = \varphi_1(x), \quad \frac{\partial T(x,\ 0)}{\partial \tau} = \varphi_2(x), \tag{2}$$

$$M_i[T(x,\ \tau)] \equiv \alpha_{1i}\frac{\partial T((i-1)l,\ \tau)}{\partial x} + \alpha_{2i}T((i-1)l,\ \tau) = 0,\ i = 1,\ 2, \tag{3}$$

where $\alpha_{1i}\alpha_{2i} = 0$; τ_r, a, and W are the relaxation time, coefficient of thermal diffusivity, and the rate of diffusion of heat. The parameters α_{1i}, α_{2i} characterize the interaction of the layer with the surrounding medium at zero temperature. The functions $\varphi_1(x)$ and $\varphi_2(x)$ are random functions of the spatial coordinates.

Using the procedure in [1], we can write the solution of the problem (1)-(3) in the form

$$T(x,\ \tau) = \sum_{n=1}^{\infty} X_n(x)\int_0^l \sum_{k=1}^{2} \Phi_{kn}(t)\,\varphi_k(x)\,X_n(x)\,dx, \tag{4}$$

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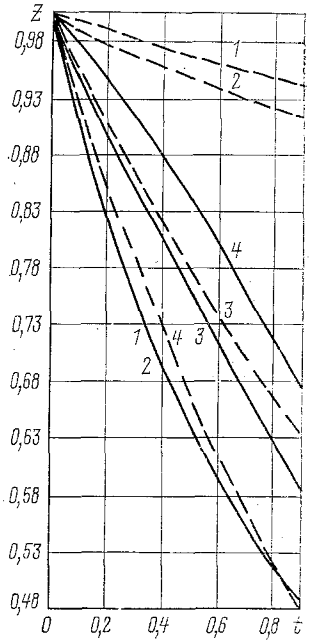

Fig. 1. Behavior of the function $z(t) \equiv K_T(t, x, x)D_0^{-1}X^{-2}(x)$: 1) steel; 2) iron; 3) aluminum; 4) copper; the continuous curves are for $\tau_r \neq 0$; the dashed curves are for $\tau_r = 0$.

where

$$\Phi_{kn}(t) = \begin{cases} \{\cos\gamma_n t + [1 + 2\tau_r(k-1)]\sin\gamma_n t\}\exp\left(-\dfrac{t}{2}\right) & \text{for } \delta_n < 0; \\ \left[(2-k)\left(1+\dfrac{t}{2}\right) + (k-1)\tau_r t\right]\exp\left(-\dfrac{t}{2}\right) & \text{for } \delta_n = 0; \\ \dfrac{1}{2}\exp\left[(k-1)\ln\dfrac{\tau_r}{\gamma_n}\right]\sum\limits_{\nu=1}^{2}(-1)^{(\nu+1)(k-1)}\times & \\ \times\exp\left[\left(-\dfrac{1}{2}-(-1)^{\nu}\gamma_n\right)t\right] & \text{for } \delta_n > 0; \end{cases}$$

$$t = \frac{\tau}{\tau_r}; \quad \gamma_n = \frac{1}{2}|\delta_n|^{1/2}; \quad \delta_n = 1 - 4P_n^2; \quad P_n = \mu_n a W^{-1};$$

$\{X_n(x)\}_{n=1}^{\infty}$ are the normalized characteristic functions of the problem

$$\frac{d^2X(x)}{dx^2} + \mu^2 X(x) = 0, \quad M_i[X(x)] = 0, \ i = 1, 2.$$

The correlation function has the form

$$K_T(t, x_1, x_2) = \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\int_0^l\int_0^l\sum_{\nu=1}^{2}\Phi_{\nu n_1}(t)\sum_{j=1}^{2}\Phi_{jn_2}(t)\,K_{\varphi_{\nu n_1}\varphi_{jn_2}}(x_1, x_2)X_{n_1}(x_1)X_{n_2}(x_2)\,dx_1dx_2X_{n_1}(x_1)X_{n_2}(x_2). \quad (5)$$

Setting $x_1 = x_2 = x$ in (5), we obtain the variance of the temperature field.

If the rate of diffusion of heat is infinite ($W = \infty$), then in view of the fact that

$$\lim_{W\to\infty}\left\{-\frac{1-\gamma_n}{2\tau_r}\right\} = -a\mu_n^2, \quad \lim_{W\to\infty}\left\{-\frac{1+\gamma_n}{2\tau_r}\right\} = -\infty,$$

Eq. (5) coincides with the corresponding results in [1].

As an example, we shall examine the random process $\varphi_1(x)$, whose correlation function has the form

$$D_0 \cos\frac{\pi x_1}{l} \cos\frac{\pi x_2}{l}, \quad D_0 = \text{const}.$$

The parameters of the problem (1)-(3) are as follows

$$\varphi_2(x) \equiv 0, \quad \alpha_{2i} \equiv 0, \; l_0 = 0.19 \cdot 10^{-6}, \; i = 1, \, 2.$$

Figure 1 shows the change in the mean-square temperature as a function of coordinates and time, where the characteristic size l_0 is chosen according to [2].

If τ_r equals the ratio of the Maxwellian relaxation time and some function of the relaxation coefficient [3], then the working equations proposed can be used without any changes to determine the corresponding probabilistic characteristics of the random multidimensional temperature fields of bounded, homogeneous, isotropic bodies. In addition, the summation must be understood as summation with respect to the natural increasing order of the Laplacian operator of the corresponding multidimensional problem.

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ONE-DIMENSIONAL MODEL OF HEAT TRANSFER IN CRYOGENIC VACUUM-SHIELD THERMAL INSULATION WITH RADIANT HEAT SOURCES

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The heat-transfer problem in an insulation consisting of layers which receive heat from external source through radiation is numerically solved in the one-dimensional approximation.

It is well known that many characteristics of modern cryogenic devices are determined by the thermal properties of the vacuum-shield thermal insulation stacks. Accordingly, more efficient new compositions of such stacks are being developed in many countries. At the same time, there is still sufficient margin for improvement left in existing vacuum-shield insulation, inasmuch as the effective thermal conductivity of these stacks in cryogenic devices is at least 1.5-3 times higher than that of the best laboratory specimens [1, 2].

According to an earlier analysis [3], one of the causes of this worsening is the presence of numerous channels running across a stack of vacuum-shield thermal insulation (gaps around the neck of the vessel, around the support rods, around the cooled object, between insulation layers, etc.) and letting hot radiation pass directly to the cold layers of the stack. Since that analysis [3] was a semiquantitative one and hardly any other studies on this subject were ever made, the authors have developed a model for calculating the heat transfer through the layers of vacuum-shield thermal insulation and taking into account the interaction of these layers with external radiant heat sources.

The main difficulties in the mathematical formulation of such a problem arise due to the intricate dependence of the thermal flux entering the insulation layers on the law of tempera-

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